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ENERGY LOSS OF A RELATIVISTIC ELECTRON IN A LORENTZIAN MODEL PLASMA

Clyde A. Morrison

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25 March 1963





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ENERGY LOSS OF A RELATIVISTIC ELECTRON IN A
LORENTZIAN MODEL PLASMA

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FOR THE COMMANDER:
Approved by

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ABSTRACT

The energy loss of a relativistic electron in a Lorentz type plasma is calculated by the method of Landau. No relativistic correction was found for a collisionless plasma; however, corrections occur for a plasma with collision losses.

1. INTRODUCTION

A fast charged particle passing through a plasma loses it energy by the excitation of plasma waves. This energy loss can be regarded macroscopically by utilizing an effective dielectric constant for the plasma. This approach was first used by E Fermi (ref 1) for a solid consisting of slightly damped electrons bound by simple harmonic forces. The approach in this paper will follow the work of Landau (ref 2) in calculating the energy loss of a relativistic electron in a plasma.

2. THEORY

Let us consider for a moment a volume of plasma large enough so that the net charge contained is zero but yet small enough so that we can average the microscopic fields over the volume and write Maxwell's equations in the average sense.

In particular these equations are

$$\nabla \cdot \vec{B}_{a} = 0$$

$$\nabla \times \vec{H}_{a} = \frac{1}{c} \vec{D}_{a}$$

$$\nabla \times \vec{E}_{a} = -\frac{1}{c} \vec{H}_{a}$$
(1)

where the subscript "a" means an average over a small volume (all equations are in c.g.s. units with $\mu=1$). We shall drop the subscripts on the fields since the only fields we will consider will be in this average sense. The above equations are incomplete in that no sources have been introduced. In the particular example, we need the charge and current density of the charged particle whose loss we are calculating. The complete equations are then

$$\nabla \cdot \vec{\mathbf{B}} = 4\pi \mathbf{q} \quad \delta(\vec{\mathbf{r}} - \vec{\mathbf{v}}t) \qquad \nabla \cdot \vec{\mathbf{B}} = \nabla \cdot \mathbf{H} = 0$$

$$\nabla \cdot \vec{\mathbf{H}} = \frac{1}{c} \frac{\partial \vec{\mathbf{D}}}{\partial t} + \frac{4\pi}{c} \mathbf{q} \vec{\mathbf{v}} \delta(\vec{\mathbf{r}} - \vec{\mathbf{v}}t) \qquad \nabla \times \vec{\mathbf{E}} = \frac{1}{c} \frac{\partial \vec{\mathbf{H}}}{\partial t}$$
(2)

where the added functions are Dirac delta functions (ref 3). The equations (2) can be expressed more simply in terms of the potentials \vec{A} and ϕ such that

$$H = \sqrt{X} A \qquad \qquad \vec{E} = -\frac{1}{c} \dot{A} - \sqrt{\psi}$$
(3)

with the gauge

$$\nabla \cdot \vec{\lambda} + \frac{1}{c} \frac{\partial (\lceil \epsilon \rceil \phi)}{\partial t} = 0$$

so that

$$\nabla^{\alpha} \vec{X} = \frac{\epsilon_{\alpha}}{\epsilon_{\alpha}} \vec{X} = -\frac{4\pi}{c} q \vec{\nabla} \delta(\vec{x} - \vec{\nabla} \epsilon)$$

$$[\epsilon] \left[\nabla^{\alpha} \phi - \frac{\epsilon_{\alpha}}{c_{\alpha}} \vec{\Phi} \right] = -4\pi q \delta(\vec{x} - \vec{\nabla} \epsilon)$$
(4)

where $[\epsilon]$ is the operator such that $\vec{D}(t) = [\epsilon] \vec{E}(t)$ and for time dependence $e^{-i\omega t}$, $[\epsilon]$ implies $\epsilon(\omega)$. We shall assume the plasma is contained in a large box with periodic boundary conditions such that we may expand all the potentials in a Fourier series as

$$\mathbf{X}(\mathbf{f},\mathbf{t}) = \sum_{\mathbf{k}} \mathbf{X}_{\mathbf{k}}(\mathbf{t}) e^{i\mathbf{K}\cdot\mathbf{f}}$$
 (5)

and

$$\varphi(\vec{x},t) = \sum_{k} \varphi_{k}(t) e^{i\vec{k}\cdot\vec{x}}$$
 (6)

The current I and charge density can be expanded the same way, so that

$$\rho = \frac{q}{V} \sum_{\mathbf{k}} e^{-i\vec{\mathbf{k}} \cdot \vec{\mathbf{v}} t} \quad e^{i\vec{\mathbf{k}} \cdot \vec{\mathbf{r}}}$$
 (7)

$$\vec{J} = \frac{q\vec{v}}{V} \sum_{k} e^{-i\vec{k} \cdot \vec{v}t} e^{i\vec{k} \cdot \vec{r}}$$
(8)

where V is the volume of the box. When these expressions are inserted into the wave equations satisfied by the two potentials we have

$$\vec{X}_{k} = \frac{\frac{4\pi}{c} \frac{q\vec{v}}{\vec{v}} e^{-1\vec{k} \cdot \vec{v}t}}{k^{2} - \frac{c(\vec{k} \cdot \vec{v})}{c^{2}} (\vec{k} \cdot \vec{v})^{2}}$$
(9)

$$\Psi_{k} = \frac{4\pi q}{V} e^{-1\vec{k} \cdot \vec{\nabla} t}$$

$$e(\vec{k} \cdot \vec{\nabla}) \left[k^{2} - \frac{e(\vec{k} \cdot \vec{\nabla})}{q^{2}} (\vec{k} \cdot \vec{\nabla})^{2}\right]$$
(10)

where we have utilised the fact that if all the fields vary as $e^{-i\vec{k}\cdot\vec{v}t}$, then [e] becomes $e(\vec{k}\cdot\vec{v})$. The electric field can be found from equation (3) and is

$$\vec{\mathbf{I}}_{\mathbf{k}} = \frac{\mathbf{i}(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}})}{c} \, \vec{\mathbf{I}}_{\mathbf{k}} - \mathbf{i} \vec{\mathbf{k}} \, \mathbf{c}_{\mathbf{k}} \tag{11}$$

Inserting the values of \vec{A}_{μ} and ϕ_{μ} in equation (11) we obtain

$$\vec{z} = \sum_{k} \vec{z}_{k} e^{i\vec{k}\cdot\vec{r}} = \frac{4\pi q}{V} \sum_{k} \frac{\frac{i\vec{k}\cdot\vec{v}}{c^{2}} \vec{v} \epsilon(\vec{k}\cdot\vec{v})^{-1}\vec{k}}{\epsilon(\vec{k}\cdot\vec{v})[k^{2} - \frac{\epsilon(\vec{k}\cdot\vec{v})}{c^{2}}(\vec{z}\cdot\vec{v})^{2}]} e^{i(\vec{k}\cdot\vec{r}-\vec{k}\cdot\vec{v}t)}$$
(12)

Now the force on the test charge is given by q (r,t) at the charge, or

$$\vec{\mathbf{y}} = \mathbf{q} \vec{\mathbf{g}} (\vec{\mathbf{r}} = \vec{\mathbf{v}} t) = \frac{14\pi \mathbf{q}^2}{V} \sum_{\mathbf{k}} \frac{(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}})}{e^{(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}})}} \vec{\mathbf{v}} \cdot e(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}}) - \vec{\mathbf{k}}$$

$$= \mathbf{q} \vec{\mathbf{g}} (\vec{\mathbf{r}} = \vec{\mathbf{v}} t) = \frac{14\pi \mathbf{q}^2}{V} \sum_{\mathbf{k}} \frac{(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}})}{e^{(\vec{\mathbf{k}} \cdot \vec{\mathbf{v}})}} (\vec{\mathbf{k}} \cdot \vec{\mathbf{v}}) - \vec{\mathbf{k}}$$
(13)

We now let the box become very large so that

$$\sum_{\mathbf{k}} = \frac{\mathbf{V}}{(2\pi)^{\mathbf{a}}} \iiint d^{\mathbf{a}} \mathbf{E}$$
 (14)

then

$$\vec{F} = \frac{14\pi q^{B}}{(2\pi)^{B}} \iiint \frac{\left[(\vec{k} \cdot \vec{v}) \frac{\vec{v}}{c^{B}} \epsilon(\vec{k} \cdot \vec{v}) - \vec{k} \right]}{\epsilon(\vec{k} \cdot \vec{v}) \left[k^{B} - \frac{\epsilon(\vec{k} \cdot \vec{v})}{c^{B}} (\vec{k} \cdot \vec{v})^{B} \right]} d^{B} \vec{k}$$
(15)

For convenience we will change variables in the integrand to

$$k_{x} = 0$$
 $k_{x} = p \cos \theta$ $k_{y} = p \sin \theta$

then

$$\mathbf{F} = \frac{\mathbf{i}\mathbf{q}^{2}}{\pi} \int_{-\infty}^{\infty} \mathbf{p} \, o \, \frac{\left[\frac{\epsilon(\mathbf{w})}{c^{2}} - \frac{1}{\mathbf{v}^{2}}\right] \, \mathbf{w} \, d\mathbf{w} \, p d\mathbf{p}}{\epsilon(\mathbf{w}) \, \left[\mathbf{p}^{2} + \mathbf{w}^{2} \, \left(\frac{1}{\mathbf{v}^{2}} - \frac{\epsilon(\mathbf{w})}{c^{2}}\right)\right]}$$
(16)

Letting $p^2v^2 = u$ and $\beta = \frac{v}{c}$ we have

$$\mathbf{F} = \frac{\mathbf{i}\mathbf{q}^3}{2\pi \mathbf{v}^3} \int_{0}^{\mathbf{p}_0} e^{3\mathbf{v}^2} d\mathbf{u} \int_{-\infty}^{\infty} \frac{\left[\beta^2 \in (\omega) - 1\right] \omega d\omega}{e(\omega) \left[\mathbf{u} + \omega^2 \left(1 - \beta^2 + e(\omega)\right)\right]}$$

3. EVALUATION OF INTEGRALS

We consider first the integral over w and let

$$I(u) = \int_{-\infty}^{\infty} \frac{\left[1 - \beta^{2} \in (\omega)\right] \omega d\omega}{\varepsilon(\omega) \left[u + \omega^{2} \left(1 - \beta^{2} \in (\omega)\right)\right]}$$
(17)

then

$$F = \frac{1q^2}{2\pi v^2} \int_0^{p_0^2 v^2} I(u) du$$

now let

$$\frac{f(\omega)}{g(\omega)} = \left[\beta^2 \in (\omega) - 1\right] \omega^2 \tag{18}$$

The integral becomes

$$I(u) = \int_{-\infty}^{\infty} \frac{-f(w) \operatorname{wdw} \beta^{2} g(w)}{[f(w) + w^{2} g(w)][ug(w) - f(w)]}$$
(19)

To evaluate this integral we shall treat ω as a complex variable and close the contour in the upper half plane. We will then have

$$I(u) + I_{c} = 2\pi i R_{+}$$
 (20)

where I is the line integral along a large semicircle and R is the residue of the integrand in the upper half plane. Now $\epsilon(\omega)^+$ approaches 1 as ω becomes very large, so that

$$I_{c} = \int \frac{w^{2}}{wdw} = \int \frac{dw}{w}$$

Let

$$\omega = Re^{i\theta}$$
 then $d\omega = Rie^{i\theta} d\theta$ (21)

and

$$I_{c} = \int_{0}^{\pi} i \ d\theta = \pi i \tag{22}$$

The integrand has one pole in the upper half plane as is shown by equation(AlO, Appendix), so that

$$R_{+} = \frac{-f(w_{o}) w_{o} \beta^{a} w g(w_{o})}{\left[f(w_{o}) + w_{o}^{a} g(w_{o})\right] \left[u \frac{dg}{dw}\right]_{w=w_{o}} \left(\frac{df}{dw}\right]_{w=w_{o}}}$$
(23)

The integral given by equation (17) becomes

$$I(u) = -\pi i -2\pi i \left[\frac{\beta^{8} f(w_{o}) g(w_{o}) w_{o}}{[f(w_{o}) + w_{o}^{2} g(w_{o})][ug'(w_{o}) - f'(w_{o})]} \right]$$
(24)

Now it is shown by equation (AlO, appendix) that the pole in the integrand is on the imaginary axis, i.e. $\omega_0=$ iy where y is now real, hence

$$I(u) = -\pi i + 2\pi i \left[\frac{\beta^{3} F(y) G(y) y}{[F(y) - y^{2} G(y)][uG'(y) - F'(y)]} \right]$$
(25)

where F(y) = f(iy), G(y) = g(iy) which are real functions as shown by equations (All) and (Al2).

The integration over u can be changed to an integration over y by using

$$uG(y) = F(y) \tag{26}$$

then

$$duG(y) = [F'(y) - uG'(y)] dy$$
 (27)

The integral over u becomes

$$F = \frac{q^2}{2v^3} \int_0^{p_0} du - \frac{q^3}{v^3} \int_y^{y_1} \frac{\beta^3 F(y) y dy}{[F(y) - y^3 G(y)]}$$
 (28)

where y is determined from $F(y_0) = 0$ and y_1 from the condition $P^{3}v^{3}G(y_1) - F(y_1) = 0$. Inserting the functions F(y) and G(y) given in equation (Al3) into the integrand we find

$$\mathbf{F} = \frac{\mathbf{q}^{2}}{2v^{2}} \int_{0}^{\mathbf{p}_{0}^{2} v^{2}} du - (1-\beta^{2}) \frac{\mathbf{q}^{2}}{v^{2}} \int_{y_{0}}^{y_{1}} y dy + \frac{\mathbf{q}^{2}}{v^{2}} \int_{y_{0}}^{y_{1}} \frac{\omega_{\mathbf{p}}^{2} y dy}{y u + \forall y + \omega_{\mathbf{p}}^{2}}$$
(29)

The first two terms are easily integrated to give

$$\frac{q^{3}}{2v^{3}} (1-\beta^{2}) \left[\left(\frac{Y}{2} \right)^{3} - \gamma \sqrt{\left(\frac{Y}{2} \right)^{2} + \frac{\beta^{3} \omega_{p}^{2}}{1-\beta^{3}}} \right]$$
 (30)

and

$$\int_{y_0}^{y_1} \frac{y \, dy}{y^2 + Yy + \omega_p^2} = \frac{1}{2} \ln \left[\frac{y_1^2 + Yy_1 + \omega_p^2}{y_0^2 + Yy_0 + \omega_p^2} \right]$$
(31)

$$-\frac{y}{\sqrt{4\omega_{p}^{2}-y^{2}}}\tan^{-1}\left[\frac{2y+y}{\sqrt{4\omega_{p}^{2}-y^{2}}}\right]_{y_{0}}^{y_{1}}$$

using the values of y_1 and y_0 given by equations (Al5) and (Al6), we have

$$F = \frac{q^{3}}{2v^{3}} (1-\beta^{3}) \left[\left(\frac{\gamma}{2} \right)^{3} - \gamma \left\{ \left(\frac{\gamma}{2} \right)^{3} + \frac{\beta^{3} \omega_{p}^{2}}{1-\beta^{3}} \right\}^{\frac{1}{2}} \right]$$

$$+ \frac{q^{3}}{2v^{3}} \omega_{p}^{3} \ln \left[\frac{v^{3} P_{o}^{3} + \gamma (1-\beta^{2})^{13} v P_{o}}{\omega_{p}^{3}} \right]$$

$$+ \frac{q^{3}}{v^{3}} \frac{\omega_{p}^{3} \gamma}{\sqrt{4\omega_{p}^{3} - \gamma^{2}}} \left[\tan^{-1} \left\{ \frac{v P_{o} + \frac{\gamma}{2} \sqrt{1-\beta^{3}}}{\sqrt{(1-\beta^{3})(\omega_{p}^{3} - \frac{\gamma}{2})^{2}}} \right\} - \tan^{-1} \left\{ \frac{\left(\frac{\gamma}{2}\right)^{3} (1-\beta^{3})\beta^{2} \omega_{p}^{3}}{\omega_{p}^{3} - \frac{\gamma}{2} \gamma} \right\} \right]$$

$$= -\frac{1}{2} \left\{ \frac{\left(\frac{\gamma}{2}\right)^{3} (1-\beta^{3})\beta^{2} \omega_{p}^{3}}{\omega_{p}^{3} - \frac{\gamma}{2} \gamma} \right\}$$

4. RESULTS

If in equation (32) we let Y=0 we find the result

$$\mathbf{F} = \frac{\mathbf{q}^2 \mathbf{w}_p^{\beta}}{\mathbf{v}^2} \ln \left(\frac{\mathbf{v} \mathbf{P}_0}{\mathbf{w}_p} \right) \tag{33}$$

which is just the energy loss per unit length of a nonrelativistic particle in a collisionless plasma. This result could be obtained directly from Maxwell's equation using only the electrostatic approximation. However, we see that when collisions are not negligible the energy loss is different from the nonrelativistic result. The leading result of equation (32) when Y is small is

$$\mathbf{F} = \mathbf{F}_{o} - \frac{\mathbf{q}^{\vee}}{2\mathbf{v}^{2}} \left[1 - \beta^{2} \right]^{1/2} \beta \omega_{p} - \frac{\mathbf{q}^{2} \gamma \omega_{p}}{2\mathbf{v}^{2}} \tan^{-1} \left[(1 - \beta^{2})^{1/2} \left(\frac{\mathbf{v}^{p}_{o}}{\mathbf{v}^{p}_{o}\beta + (1 - \beta^{2})\omega_{p}} \right) \right]$$
(34)

and if we assume β not small $[\beta v P_0 >> (1-\beta^8)w_D]$ the correction becomes

$$-\frac{q^{2}\gamma w_{p}}{2v^{2}}\left[\beta(1-\beta^{2})^{12}+\tan^{-1}\sqrt{\frac{1-\beta^{2}}{\beta^{2}}}\right]$$
(35)

so that the energy loss is less for small Y.

Throughout the derivation the cutoff parameter P has been assumed large, that is, vP has been assumed large compared to $\mathfrak m$ and to Y. In using equation (32) for computation the choice of P sust be consistent with this assumption. For a plasma in terral equilibrium one may choose $P_0 \approx \frac{1}{2}$ where λ_D is the Debye length in the plasma (ref 4). This result would only give the loss of the test particle up to this value P. One would then have to calc ulate the loss by another method and add to the result above to give the complete loss. For an electron gas not in thermal equilibrium a reasonable estimate of P_0 would be n where n is the number of free electrons per cubic centimeter. For values of P greater than this, the individual electron scattering would be important; and if this were calculated, one could again obtain the total loss by simply adding to the result given here.

ACKNOWLEDGMENT

The author wishes to thank N. Karayianis for helpful suggestions in the derivation of equation (29).

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APPENDIX

Derivation of Dielectric Constant and the Evaluation of the Poles in the Integrand of Equation 19.

We take for our model of a plasma a simple Lorentz-type gas. We assume that we have a number (n) of free electrons neutralized by a continuous background of positive charge. The force on an electron then is given by

$$\vec{n} = -\vec{n} - \vec{n} \vec{r} \tag{A1}$$

where Y is a loss parameter which may be interpreted as energy loss by collision with other electrons or the positive background, and \vec{r} is the displacement of an electron from equilibrium. Now since $\vec{y} = -n \cdot \vec{r}$, we may write

$$\frac{\mathbf{n}^{\mathbf{p}}}{\mathbf{n}\mathbf{e}} = \mathbf{e}\mathbf{I} - \frac{\mathbf{n}\mathbf{v}\mathbf{p}}{\mathbf{n}\mathbf{e}} \tag{A2}$$

or

$$\ddot{\vec{p}} = \frac{ne^3}{n} \vec{z} - \gamma \dot{\vec{p}} \tag{A3}$$

If we now assume a time dependence e -iwt then

$$\vec{P} = \frac{-\frac{ne^2}{m}}{m(m+i\gamma)} \tag{A4}$$

and since $\vec{D} = \vec{x} + 4\pi \vec{P} = \epsilon \vec{x}$ we have

$$D = \left[1 - \frac{4\pi ne^2}{\omega(\omega + i\gamma)}\right] E$$
 (A5)

where

$$\epsilon = 1 - \frac{w_p^s}{w(w+i\gamma)}$$
 and $w_p^s = \frac{4\pi me^s}{m}$

If we substitute this expression for ϵ in equation (18) we have

$$f(w) = [(\beta^2-1) w^2 + i Y (\beta^2-1) w - \beta^2 w_n^2]w$$
 (A6)

and

$$g(w) = \frac{w + i\gamma}{w} \tag{A7}$$

The poles in the integrand of equation (19) are then the zeros of

$$\omega[ug(\omega) - f(\omega)] = u[\omega + i \gamma] + \omega[(1-\beta^2)\omega^2 + i\gamma(1-\beta^2)\omega + \beta^2\omega_p^2]$$
 (A8)

$$ug(w) - f(w) = (1-\beta^2) w^3 + i\gamma(1-\beta^2) w^3 + (u+\beta^2 w_p^2) w + i\gamma u = 0$$
 (A9)

Let w = iZ, then

$$(1-\beta^2)Z^3 + Y(1-\beta^2)Z^2 - (u+\beta^2 \omega_0^2)Z - Yu = 0$$
 (A10)

This equation has three real roots, so that the roots in w are all on the imaginary axis. Further, the equation has only one positive and two negative roots. This is easily seen by applying Descartes Rule of Signs to the cubic equation (4). Now if we let this root be iy, we have

$$\mathbf{F}(y) = f(iy) = [(1-\beta^2) y^2 + \gamma (1-\beta^2) y - \beta^2 \omega_p^2]$$
 (All)

$$G(y) = g(iy) = \frac{y+\gamma}{y}$$
 (A12)

and the function
$$F(y) - y^2G(y) = -\beta^2 \left[y^2 + \gamma y + \omega_p^2 \right]$$
 (A13)

The limits on the integral in equation (29) are then given

$$y_0 = -\frac{Y}{2} + \sqrt{(\frac{Y}{2})^2 + \frac{\beta^2 \omega_p^2}{1-\beta^2}}$$
 (A14)

and

$$y_1 = \sqrt{\frac{u + \beta^2 w_p^2}{1 - \beta^2}}$$
 (A15)

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